Having taken care of the case where the characteristic polynomial of a linear homogeneous constant coefficient 2nd order ODE has complex roots, we turn to the double root case.
We recognize this case by the fact that the characteristic polynomial is the stare of a linear binomial $(r - r_1)^2$ Therefore we know one solution to the ODE: $y_1 = e^{r_1 t}$ but are at loss for what to do for the second y_2 which is needed to form a fundamental pair. We actually already have the machinery in place to deal with this; namely Abel's formula for the Wronskian of such an ODE gives us a way to find a second solution, if one solution $y_1 = e^{r_1 t}$ is known.
Let us illustrate this for the following ODE: $y'' + 2y' + y = 0$. According to Abel's formula the Wronskian is given by $W = e^{\int 2\,dt} = e^{2t+C} = C_1 e^{2t}$, and we assume that $C_1 = 1$ for the sake of simplicity. Therefore

$$W(e^t, y_2)(t) = \det \begin{pmatrix} e^t & y_2 \\ e^t & y_2' \end{pmatrix} = e^{2t}$$

This produces that following linear first order ODE for y_2

$$e^t y_2' - e^t y_2 = e^{2t} \qquad \text{or} \qquad y_2' - y_2 = e^t$$

An integrating factor for this linear first order ODE is $\mu = \int -1\,dt = e^{-t}$. Therefore,

$$(y_2 e^{-t})' = 1$$

and consequently

$$y_2 e^{-t} = t + C$$

From which we see that $y_2 = te^t + Ce^t$. Since we already know that e^t solves our ODE we my add $-Ce^t$ to y_2 (i.e., discard Ce^t). We now see that $y_2 = te^t$ is the a second solution to the ODE. Since we already know the Wronskian $W(y_1, y_2) = e^2 t$ is never zero, we actually have a fundamental pair y_1, y_2. If you wish to double check you can look up a previous FAQ which asked to verify this simple fact by actually evaluating the determinant.
The general solution should be rewritten in the economical form $y = e^t(c_1 + c_2 t)$ and for the sake of solving IVP's we should also write $y' = y + c_2 e^t$. Therefore if the IVP is $y(0) = 34, \quad y'(0) = 37$, then $c_1 = 34$ and $c_2 = 3$.
It maybe helpful to use maxima to plot solution to a number of second order constant coefficient linear homogeneous ODE's to see the various types of solutions that can be produced. For example to see decaying oscillation one would enter the following instructions under xMaxima:

(%i1) eqn: 16*'diff$(y, t, 2)$ + 8*'diff$(y, t) + 257 * y$;
(%i2) sln: ode2(eqn, y, t);
(%i3) sln1:ic2(sln, $t = 0, y = 1$,diff$(y, t) = 1$);
(%i4) plot2d(rhs(sln1,[$t, -2, 50$]);