Once we have a collection of functions for which we have found Fourier series, there are two important shortcuts to finding Fourier series of new functions: the first is algebra and the second is term by term integration.
Before we illustrate these, we need to emphasize that without further restrictions term by term differentiate of Fourier series is not a legitimate process. This is in sharp contrast with power series, covered in Math 140, much of whose significance stems from the repeated term by term differentiation which can be performed on them. To see what can go wrong if one differentiates a Fourier series consider the Fourier series of $f(x) = x$ on the interval $[-\pi, \pi]$ which was on the last FAQ:

$$x = \sum_{n=1}^{\infty}(-1)^{n+1}\frac{2}{n}\sin nx = \frac{2}{1}\sin x - \frac{2}{2}\sin 2x + \frac{2}{3}\sin 3x + \dots$$

If we differentiate both sides of the above formula we get

$$1 = \sum_{n=1}^{\infty}(-1)^{n+1}2\cos nx = 2\cos x - 2\cos 2x + 2\cos 3x + \dots$$

This obviously makes no sense. For example set $x = 0$, $x = \pi/6$, $x = \pi/4$, etc, if you have any doubts.
Now suppose you wish to find the Fourier series of

$$g(x) \quad = \quad \begin{cases} -1, & \text{if} \quad x < 0 \\ 1, & \text{if} \quad 0 \le x \end{cases}$$

on $[-2, 2]$.
Since we already found the Fourier series $u(x)$ on $[-2, 2]$ last time:

$$\frac{1}{2} + \sum_{1}^{\infty} \frac{1}{n\pi}(1 - \cos n\pi)\sin\left(\frac{n\pi}{2}x\right)$$

and since $g(x) = 2u(x) - 1$ we simply multiply the above by 2 and subtract one to obtain:

$$g(x) = \sum_{n=1}^{\infty}\frac{2}{n\pi}(1 - \cos(n\pi))\sin\left(\frac{n\pi}{2}x\right)$$

Suppose we wish to find the Fourier series of $f(x) = |x|$ on on $[-2, 2]$. We note that the derivative of $f(x)$ is $g(x)$ and proceed to take the indefinite integral of the Fourier series of $g(x)$ term by term:

$$f(x) = C + \sum_{n=1}^{\infty}\frac{-4}{n^2\pi^2}(1 - \cos(n\pi))\cos\left(\frac{n\pi}{2}x\right)$$

The C represents the constant of integration since we applied indefinite integration to both sides. The C can be evaluated very easily if we recall that $\frac{a_0}{2}$ belongs in the position that C occupies and that the value of a_0 is the area underneath the graph divided by L. That is, a_0 is $\frac{4}{2}$ and hence $C = 1$.
Now let us consider a problem that can easily be solved by term by term integration. Find the Fourier series $h(x) = x^2$ on on $[-\pi, \pi]$. Therefore we expect that applying applying by term integration to the Fourier series of x on $[-\pi, \pi]$ will give us the answer:

$$\frac{1}{2}x^2 = C + \sum_{n=1}^{\infty}(-1)^n\frac{2}{n^2}\cos nx = -\frac{2}{1^2}\cos x + \frac{2}{2^2}\cos 2x - \frac{2}{3^2}\cos 3x + \dots$$

Unfortunately, the integration produced a constant of integration since we took indefinite integrals. We recognize it must be $\frac{a_0}{2}$ where a_0 is the zero-th Fourier coefficient of the function $\frac{1}{2}x^2$ on $[-\pi, \pi]$. This a_0 is easily found using the formula

$$a_0 = \frac{1}{\pi}\int_{\pi}^{\pi}\frac{1}{2}x^2\,dx = \frac{1}{\pi}\int_{0}^{\pi}x^2\,dx = \frac{1}{3\pi}\left[x^3\right]_0^{\pi} = \frac{1}{3}\pi^2$$

Finally, we arrive at

$$x^2 = \frac{1}{3}\pi^2 + \sum_{n=1}^{\infty}(-1)^n\frac{4}{n^2}\cos nx = -\frac{4}{1^2}\cos x + \frac{4}{2^2}\cos 2x - \frac{4}{3^2}\cos 3x + \dots$$

Although this seems like a long procedure, one should compare it with the process of computing the Fourier series directly from the standard formulas which involve double integration by parts.